

Third Prize Winner Mr. Peeyush Ranjan Pande's Solution

Given:

$$BM = MA$$

$$AE \perp CB$$

$$BL = LC$$

$$DF = \frac{1}{2} AE$$

To Prove:

$$\angle MFL = \angle B$$

Construction:

Join MD, MF, FL & BE

Proof:

$$\text{As } DF = \frac{1}{2} AE$$

$$\Rightarrow AF + DE = AE - FD = \frac{1}{2} AE$$

$$\therefore AF + ED = FD$$

Let K on FD such that

$$AF = FK \text{ \& } KD = DE$$

Join BK and ML

$\triangle ABD$ right triangle

DM is the median

$$DM = BM = AM$$

$$\Rightarrow \angle MDB = \angle B$$

M is midpoint of AB

F is midpoint of AK

\therefore By midpoint theorem

$$MF \parallel BK$$

$$\angle ACB = \angle AEB \text{ (same chord)}$$

$$= \angle C$$

In $\triangle BDK, \triangle BDE$

$$BD = BD$$

$$\angle BDK = \angle BDE$$

$$DK = DE$$

$$\therefore \triangle BDK \cong \triangle BDE$$

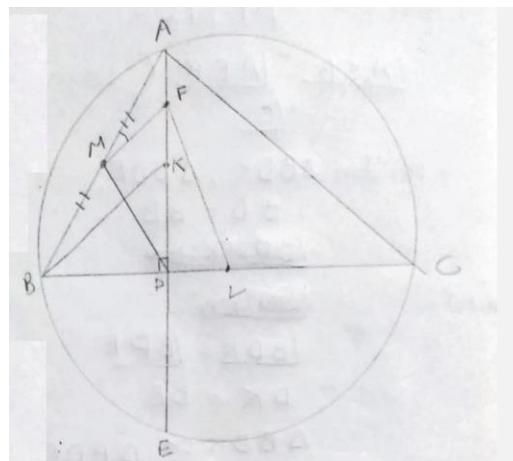
$$\angle BKE = \angle BED = \angle C$$

$$BK \parallel MF$$

$$\angle BKE = \angle MFE = \angle C$$

$$\angle BAD = 90^\circ - \angle B$$

$$\therefore \angle AMF = \angle C - (90^\circ - \angle B)$$



$$= \angle C + \angle B - 90^\circ$$

$$= \angle A + \angle B + \angle C - 90^\circ - \angle A$$

$$= 180^\circ - 90^\circ = \angle A$$

$$= 90^\circ - \angle A$$

M is midpoint of AB, L is midpoint of BC

\therefore By MPT, $ML \parallel AC$

$$\angle LMB = \angle CAB = \angle A$$

$$\angle FML = 180^\circ - \angle A - (90^\circ - A)$$

$$= 90^\circ$$

$$= \angle FDL$$

FMDL is cyclic quadrilateral

$$\therefore \angle MDB = \angle MFL$$

(Exterior \angle = interior angle)

$$\angle MDB = \angle MBD = \angle B$$

$$\therefore \angle B = \angle MFL$$